Lösungen zu
Aufgaben zu Winkel zwischen Vektoren (3)

Winkel Berechnung

a) $|\vec{a}|=\sqrt{2^2+(2\cdot\sqrt{3})^2}=4$

$|\vec{b}|=\sqrt{(3\cdot\sqrt{3})^2+3^2}=6$

$$\frac{\vec{a}\circ\vec{b}}{|\vec{a}|\cdot|\vec{b}|}=\frac{\begin{pmatrix}2\\2\cdot\sqrt{3}\end{pmatrix}\circ\begin{pmatrix}3\cdot\sqrt{3}\\3\end{pmatrix}}{4\cdot 6}=\frac{12\cdot\sqrt{3}}{24}=\frac{\sqrt{3}}{2}$$

$\Rightarrow\ \alpha=\arccos\left(\frac{\sqrt{3}}{2}\right)=30°$

b) $|\vec{a}|=\sqrt{(-4)^2+(4\cdot\sqrt{3})^2}=8$

$|\vec{b}|=\sqrt{\left(\frac{1}{4}\right)^2+\left(\frac{\sqrt{3}}{4}\right)^2}=\frac{1}{2}$

$$\frac{\vec{a}\circ\vec{b}}{|\vec{a}|\cdot|\vec{b}|}=\frac{\begin{pmatrix}-4\\4\cdot\sqrt{3}\end{pmatrix}\circ\begin{pmatrix}\frac{1}{4}\\\frac{\sqrt{3}}{4}\end{pmatrix}}{8\cdot\frac{1}{2}}=\frac{-1+3}{4}=\frac{1}{2}$$

$\Rightarrow\ \alpha=\arccos\left(\frac{1}{2}\right)=60°$

c) $|\vec{a}|=\sqrt{\left(-\frac{5\cdot\sqrt{3}}{2}\right)^2+\left(-\frac{5}{2}\right)^2}=5$

$|\vec{b}|=\sqrt{0^2+7^2}=7$

$$\frac{\vec{a}\circ\vec{b}}{|\vec{a}|\cdot|\vec{b}|}=\frac{\begin{pmatrix}-\frac{5\cdot\sqrt{3}}{2}\\-\frac{5}{2}\end{pmatrix}\circ\begin{pmatrix}0\\7\end{pmatrix}}{5\cdot 7}=\frac{-\frac{35}{2}}{35}=-\frac{1}{2}$$

$\Rightarrow\ \alpha=\arccos\left(-\frac{1}{2}\right)=120°$

d) $|\vec{a}|=\sqrt{(\sqrt{3})^2+1^2}=2$

$|\vec{b}|=\sqrt{(\sqrt{6}+\sqrt{2})^2+(\sqrt{6}-\sqrt{2})^2}$

$=\sqrt{6+4\cdot\sqrt{3}+2+6-4\cdot\sqrt{3}+2}=4$

$$\frac{\vec{a}\circ\vec{b}}{|\vec{a}|\cdot|\vec{b}|}=\frac{\begin{pmatrix}\sqrt{3}\\1\end{pmatrix}\circ\begin{pmatrix}\sqrt{6}+\sqrt{2}\\\sqrt{6}-\sqrt{2}\end{pmatrix}}{2\cdot 4}$$

$$=\frac{3\cdot\sqrt{2}+\sqrt{6}+\sqrt{6}-\sqrt{2}}{8}$$

$$=\frac{2\cdot\sqrt{2}+2\cdot\sqrt{6}}{8}$$

$$=\frac{\sqrt{2}+\sqrt{6}}{4}$$

$\Rightarrow\ \alpha=\arccos\left(\frac{\sqrt{2}+\sqrt{6}}{4}\right)=15°$

Vektoren Bestimmen

a)

$$\cos(60°)=\frac{1}{2}$$
$$|\vec{a}|=\sqrt{(\sqrt{3})^2+1^2}=2$$
$$|\vec{b}|=\sqrt{b_1^2+b_2^2}=1 \Rightarrow b_1^2+b_2^2=1 \Rightarrow b_2=\pm\sqrt{1-b_1^2}$$
$$\frac{1}{2}=\frac{\begin{pmatrix}\sqrt{3}\\1\end{pmatrix}\circ\begin{pmatrix}b_1\\b_2\end{pmatrix}}{2}$$
$$\Leftrightarrow\ 1=\sqrt{3}\cdot b_1+b_2$$
$$\Rightarrow\ 1=\sqrt{3}\cdot b_1\pm\sqrt{1-b_1^2} \Rightarrow b_1=0 \wedge b_2=1$$
$$\vec{b}=k\cdot\begin{pmatrix}0\\1\end{pmatrix}\ ;\ k\in\mathbb{R}$$

b)

$$\cos(150°)=-\frac{\sqrt{3}}{2}$$
$$|\vec{a}|=\sqrt{(-\sqrt{3})^2+(-1)^2}=2$$
$$|\vec{b}|=\sqrt{b_1^2+b_2^2}=1 \Rightarrow b_1^2+b_2^2=1 \Rightarrow b_2=\pm\sqrt{1-b_1^2}$$
$$-\frac{\sqrt{3}}{2}=\frac{\begin{pmatrix}-\sqrt{3}\\-1\end{pmatrix}\circ\begin{pmatrix}b_1\\b_2\end{pmatrix}}{2}$$
$$\Leftrightarrow\ -\sqrt{3}=-\sqrt{3}\cdot b_1-b_2$$
$$\Rightarrow\ -\sqrt{3}=-\sqrt{3}\cdot b_1\pm\sqrt{1-b_1^2} \Rightarrow b_1=0 \wedge b_2=\pm1$$
$$\vec{b}=k\cdot\begin{pmatrix}1\\0\end{pmatrix}\ ;\ k\in\mathbb{R}$$

Aussagen Überprüfen

a) richtige

b) falsch

c) richtig

d) falsch

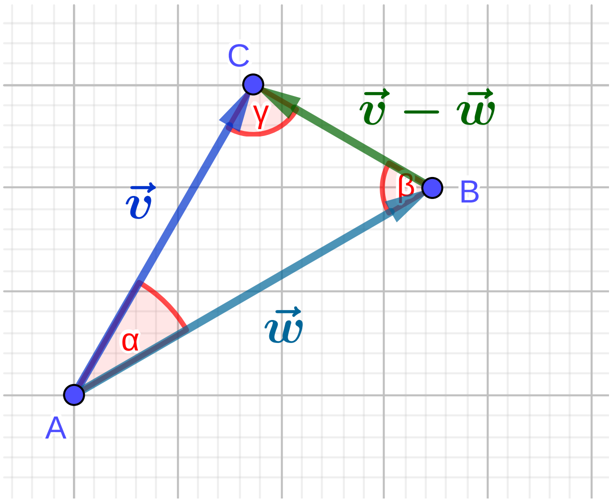
e) falsch

f) richtig

g) richtig

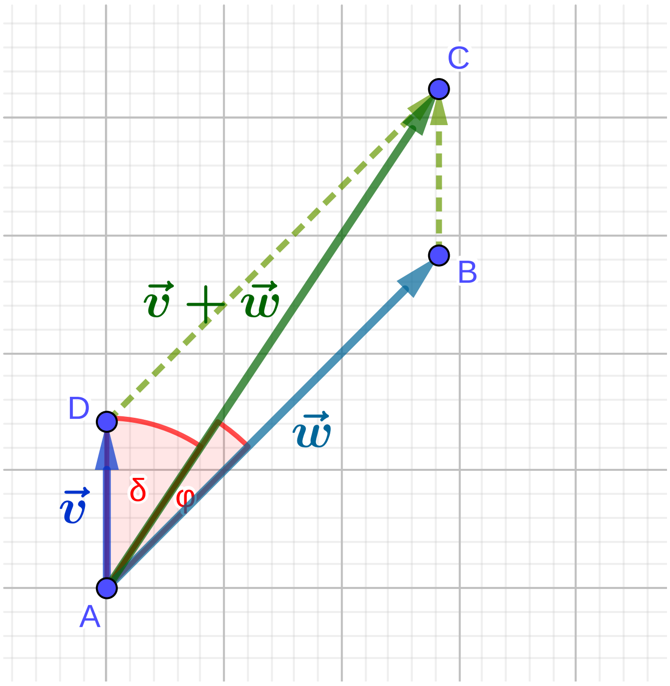
Geometrische Figuren

a) Schaubild:



$$|\vec{v}|=\sqrt{(\sqrt{3})^2+3^2}=2\cdot\sqrt{3}$$
$$|\vec{w}|=\sqrt{(2\cdot\sqrt{3})^2+2^2}=4$$
$$\cos(\alpha)=\frac{\vec{v}\circ\vec{w}}{|\vec{v}|\cdot|\vec{w}|}=\frac{\begin{pmatrix}\sqrt{3}\\3\end{pmatrix}\circ\begin{pmatrix}2\cdot\sqrt{3}\\2\end{pmatrix}}{8\cdot\sqrt{3}}=\frac{6+6}{8\cdot\sqrt{3}}=\frac{3}{2\sqrt{3}}$$
$$\Rightarrow\ \alpha=\arccos\left(\frac{3}{2\sqrt{3}}\right)=30°$$
$$|\vec{v}-\vec{w}|=\left|\begin{pmatrix}-\sqrt{3}\\1\end{pmatrix}\right|=\sqrt{(-\sqrt{3})^2+1^2}=2$$
$$\cos(\beta)=\frac{(\vec{v}-\vec{w})\circ\vec{w}}{|\vec{v}-\vec{w}|\cdot|\vec{w}|}=\frac{\begin{pmatrix}-\sqrt{3}\\1\end{pmatrix}\circ\begin{pmatrix}2\cdot\sqrt{3}\\2\end{pmatrix}}{2\cdot4}=\frac{-6+2}{8}=\frac{4}{8}=\frac{1}{2}$$
$$\Rightarrow\ \beta=\arccos\left(\frac{1}{2}\right)=60°$$
$$\gamma=180°-\alpha-\beta=90°$$

b) Schaubild:



$$|\vec{v}|=\sqrt{2}$$

$$|\vec{w}|=\sqrt{2\cdot(2\cdot\sqrt{2})^2}=4$$

$$|\vec{v}+\vec{w}|=\left|\begin{pmatrix}2\cdot\sqrt{2}\\3\cdot\sqrt{2}\end{pmatrix}\right|=\sqrt{(2\cdot\sqrt{2})^2+(3\cdot\sqrt{2})^2}=\sqrt{8+18}=\sqrt{26}$$

$$\cos(\delta)=\frac{\vec{v}\circ(\vec{v}+\vec{w})}{|\vec{v}|\cdot|(\vec{v}+\vec{w})|}=\frac{\begin{pmatrix}0\\\sqrt{2}\end{pmatrix}\circ\begin{pmatrix}2\cdot\sqrt{2}\\3\cdot\sqrt{2}\end{pmatrix}}{\sqrt{2}\cdot\sqrt{26}}=\frac{6}{\sqrt{52}}$$

$$\Rightarrow\ \delta=\arccos\left(\frac{6}{\sqrt{26}}\right)\approx 33{,}69\,°$$

$$\cos(\varphi)=\frac{\vec{w}\circ(\vec{v}+\vec{w})}{|\vec{w}|\cdot|(\vec{v}+\vec{w})|}=\frac{\begin{pmatrix}2\cdot\sqrt{2}\\2\cdot\sqrt{2}\end{pmatrix}\circ\begin{pmatrix}2\cdot\sqrt{2}\\3\cdot\sqrt{2}\end{pmatrix}}{4\cdot\sqrt{26}}=\frac{8+12}{4\cdot\sqrt{26}}=\frac{5}{\sqrt{26}}$$

$$\Rightarrow\ \varphi=\arccos\left(\frac{5}{\sqrt{26}}\right)\approx 11{,}31\,°$$

$$\frac{\varphi}{\delta}\approx\frac{33{,}69\,°}{11{,}31\,°}\approx 2{,}98$$