Lösungen zu
Aufgaben zu Winkel zwischen Vektoren (2)

Winkel Berechnung

a) $|\vec{a}|=\sqrt{(\sqrt{8})^2+(\sqrt{8})^2}=4$

$|\vec{b}|=\sqrt{1^2+0^2}=1$

$$\frac{\vec{a}\circ\vec{b}}{|\vec{a}|\cdot|\vec{b}|}=\frac{\begin{pmatrix}\sqrt{8}\\ \sqrt{8}\end{pmatrix}\circ\begin{pmatrix}1\\ 0\end{pmatrix}}{4\cdot 1}=\frac{1}{\sqrt{2}}$$

$$\Rightarrow\ \alpha=\arccos\left(\frac{1}{\sqrt{2}}\right)=45\,°$$

$|\vec{a}|=\sqrt{(\sqrt{3})^2+1^2}=2$

$|\vec{b}|=\sqrt{(\sqrt{3})^2+0^2}=\sqrt{3}$

b) $$\frac{\vec{a}\circ\vec{b}}{|\vec{a}|\cdot|\vec{b}|}=\frac{\begin{pmatrix}\sqrt{3}\\ 1\end{pmatrix}\circ\begin{pmatrix}\sqrt{3}\\ 0\end{pmatrix}}{2\cdot\sqrt{3}}=\frac{3}{2\cdot\sqrt{3}}=\frac{\sqrt{3}}{2}$$

$$\Rightarrow\ \alpha=\arccos\left(\frac{\sqrt{3}}{2}\right)=30\,°$$

c) $|\vec{a}|=\sqrt{\left(-\frac{\sqrt{3}}{2}\right)^2+\left(\frac{3}{2}\right)^2}=\sqrt{3}$

$|\vec{b}|=\sqrt{2^2+0^2}=2$

$$\frac{\vec{a}\circ\vec{b}}{|\vec{a}|\cdot|\vec{b}|}=\frac{\begin{pmatrix}-\frac{\sqrt{3}}{2}\\ \frac{3}{2}\end{pmatrix}\circ\begin{pmatrix}2\\ 0\end{pmatrix}}{2\cdot\sqrt{3}}=\frac{-\sqrt{3}}{2\cdot\sqrt{3}}=-\frac{1}{2}$$

$$\Rightarrow\ \alpha=\arccos\left(-\frac{1}{2}\right)=120\,°$$

d) $|\vec{a}|=\sqrt{3^2+2{,}64^2}\approx 4$

$|\vec{b}|=\sqrt{\left(\frac{1}{2}\right)^2+0^2}=\frac{1}{2}$

$$\frac{\vec{a}\circ\vec{b}}{|\vec{a}|\cdot|\vec{b}|}=\frac{\begin{pmatrix}3\\ 2{,}64\end{pmatrix}\circ\begin{pmatrix}\frac{1}{2}\\ 0\end{pmatrix}}{4\cdot\frac{1}{2}}=\frac{\frac{3}{2}}{2}=-\frac{3}{4}$$

$$\Rightarrow\ \alpha=\arccos\left(\frac{3}{4}\right)\approx 41{,}4\,°$$

e) $|\vec{a}|=\sqrt{2{,}35^2+0^2}=2{,}35$

$|\vec{b}|=\sqrt{(-5)^2+12^2}=13$

$$\frac{\vec{a}\circ\vec{b}}{|\vec{a}|\cdot|\vec{b}|}=\frac{\begin{pmatrix}2{,}35\\ 0\end{pmatrix}\circ\begin{pmatrix}-5\\ 12\end{pmatrix}}{2{,}35\cdot 13}=\frac{-11{,}75}{30{,}55}\approx -0{,}3846$$

$$\Rightarrow\ \alpha\approx\arccos(-0{,}3846)\approx 112{,}6\,°$$

Vektoren Bestimmen

a) $\vec{a} \approx \begin{pmatrix} 2 \\ \pm 4{,}6 \end{pmatrix}$

b) $\vec{a} = \begin{pmatrix} \sqrt{6}+\sqrt{2} \\ \pm(\sqrt{6}-\sqrt{2}) \end{pmatrix}$

c) $\vec{a} \approx \begin{pmatrix} 0{,}45 \\ \pm 0{,}22 \end{pmatrix}$

d) $\vec{a} = \begin{pmatrix} -\frac{\sqrt{6}+\sqrt{2}}{2} \\ \pm\frac{\sqrt{6}-\sqrt{2}}{2} \end{pmatrix}$