Lösung 5

$$\begin{aligned}(x+h)^n &= \binom{n}{0}x^n+\binom{n}{1}h\,x^{n-1}+\binom{n}{2}h^2x^{n-2} \\ &\quad \cdots +\binom{n}{n-2}h^{n-2}x^2+\binom{n}{n-1}h^{n-1}x+\binom{n}{n}h^n \\ &= x^n+\left(\sum_{k=0}^{n}\binom{n}{k}h^k x^{n-k}\right)+h^n\end{aligned}$$

Lösung 4

$$a_{n+1,k}=a_{n,k-1}+a_{n,k} \underset{\text{nach Voraussetzung}}{=} \binom{n}{k-1}+\binom{n}{k}=\binom{n+1}{k}$$

Lösung 3

$$a_{n+1,k}=a_{n,k-1}+a_{n,k}\,,\quad 1<k\le n$$

Lösung 2

$$a_{0,0}=1=\binom{0}{0},\quad a_{1,0}=1=\binom{1}{0},\quad a_{1,1}=1=\binom{1}{1}$$

Lösung 1

$$\begin{aligned}\binom{n}{k-1}+\binom{n}{k} &= \frac{n!}{(n-(k-1))!(k-1)!}+\frac{n!}{(n-k)!k!} \\ &= \frac{n!}{(n-k+1)!(k-1)!}+\frac{n!}{(n-k)!k!} \\ &= \frac{n!}{(n-k+1)(n-k)!(k-1)!}+\frac{n!}{(n-k)!k(k-1)!} \\ &= \frac{k\,n!}{(n-k+1)(n-k)!k(k-1)!}+\frac{(n-k+1)n!}{(n-k+1)(n-k)!k(k-1)!} \\ &= \frac{(k+n-k+1)n!}{(n-k+1)(n-k)!k(k-1)!} \\ &= \frac{(n+1)n!}{(n-k+1)(n-k)!k(k-1)!} \\ &= \frac{(n+1)!}{(n+1-k)!k!} \\ &= \binom{n+1}{k}\end{aligned}$$

Fakultät

$$n!=n\cdot(n-1)\cdot(n-2)\cdots3\cdot2\cdot1=n\cdot(n-1)!\;;\quad 0!=1$$

Definition Binomialkoeffizient

$$\binom{n}{k}=\frac{n!}{(n-k)!k!}$$

Momentane Änderungsrate (4)